

How to Solve a Minecraft Puzzle Like a Mathematician

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Outline (MCC 15)



① The Puzzle

② Context

③ Helpful Properties

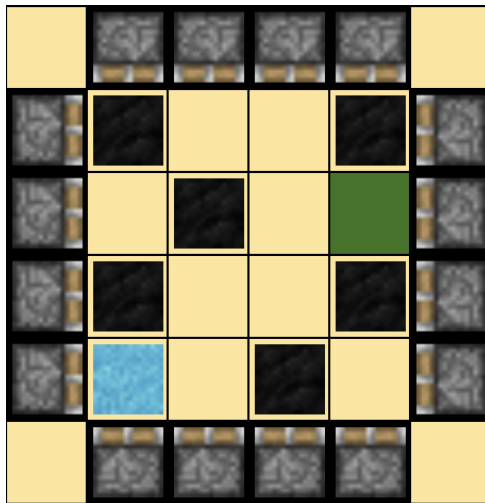
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④ The 'Best' Strategy?

⑤ Conclusion



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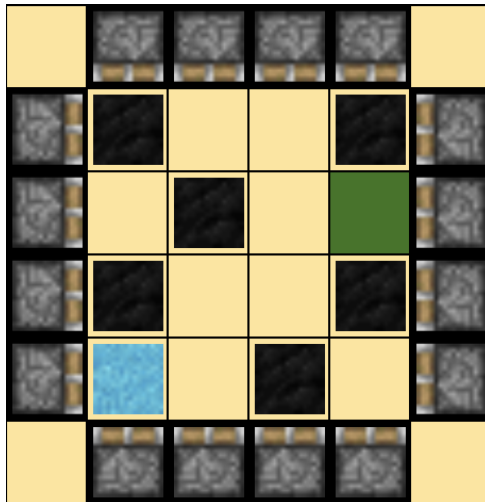
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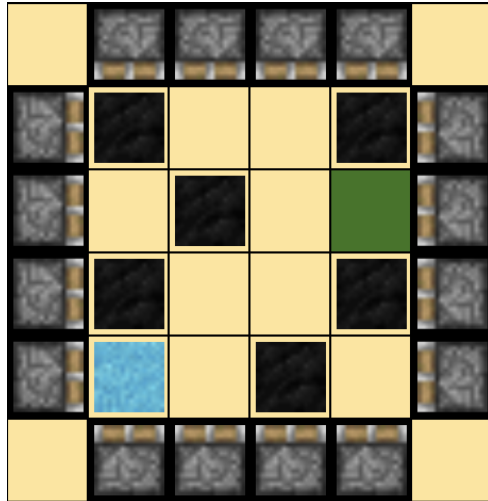
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A Simple Example (MCC 15)

- **Objective:** Get the blue block to the goal (green space).



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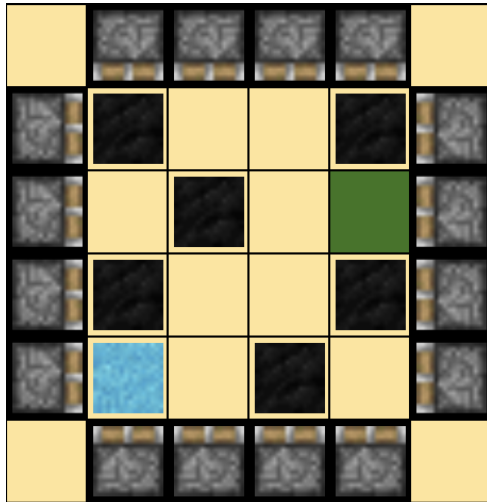
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A Simple Example (MCC 15)

- **Objective:** Get the blue block to the goal (green space).
- The pistons can push any blocks in front of them.



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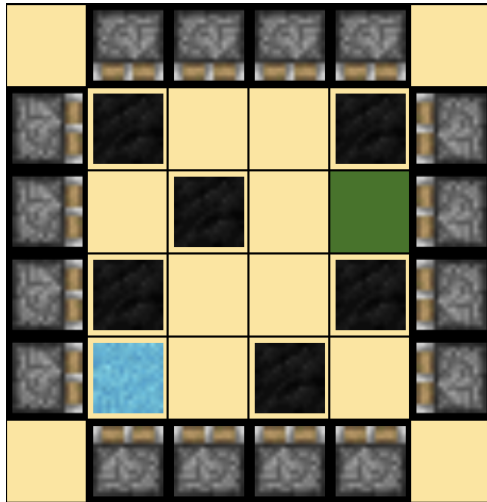
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A Simple Example (MCC 15)

- **Objective:** Get the blue block to the goal (green space).
- The pistons can push any blocks in front of them.
- Once we activate a piston, we'll immediately retract it.



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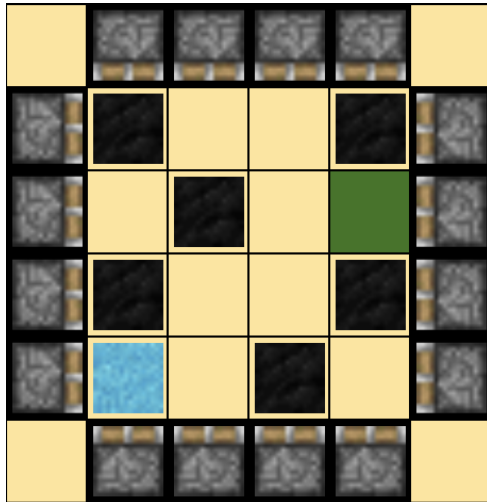
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- Once we activate a piston, we'll immediately retract it.
- Solution:



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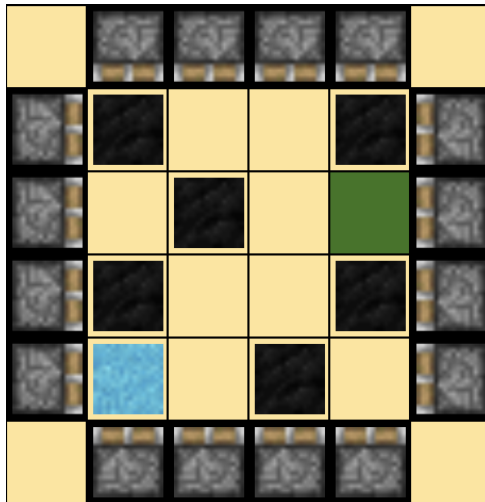
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A Simple Example (MCC 15)

- **Objective:** Get the blue block to the goal (green space).
- The pistons can push any blocks in front of them.
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- Solution:
 $1 \rightarrow, B \uparrow, 2 \rightarrow, C \uparrow, A \downarrow, 3 \rightarrow.$



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Minecraft Minigame: Sands of Time



- The puzzle is at the beginning of a vast maze-like dungeon.

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Minecraft Minigame: Sands of Time



- The puzzle is at the beginning of a vast maze-like dungeon.
- Not much time to explore!

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Minecraft Minigame: Sands of Time



- The puzzle is at the beginning of a vast maze-like dungeon.
- Not much time to explore!
- This puzzle is attempted when there's not much else to do.

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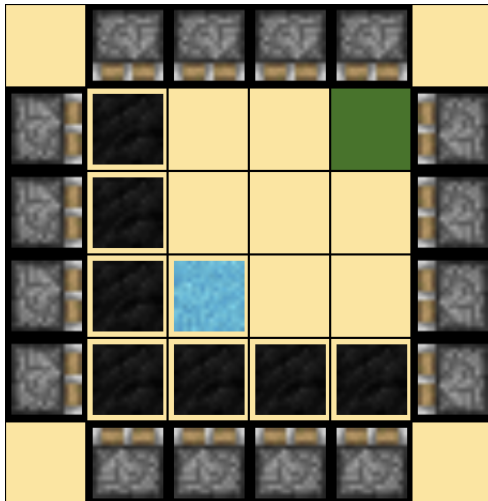
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Objective (MCC 16)

Our strategy should be:



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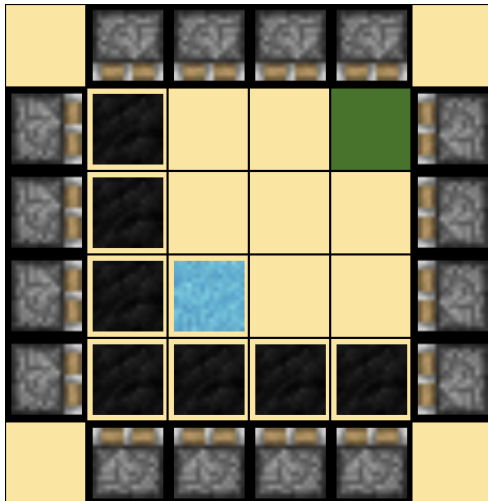
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Objective (MCC 16)

Our strategy should be:

- Not too hard to learn



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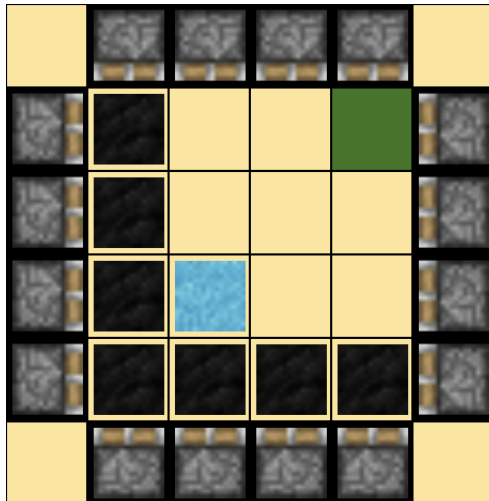
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Objective (MCC 16)

Our strategy should be:

- Not too hard to learn
- Easy to execute



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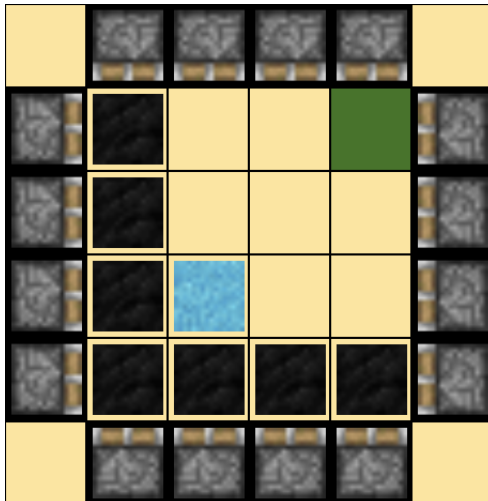
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Objective (MCC 16)



Our strategy should be:

- Not too hard to learn
- Easy to execute
- Quick to execute



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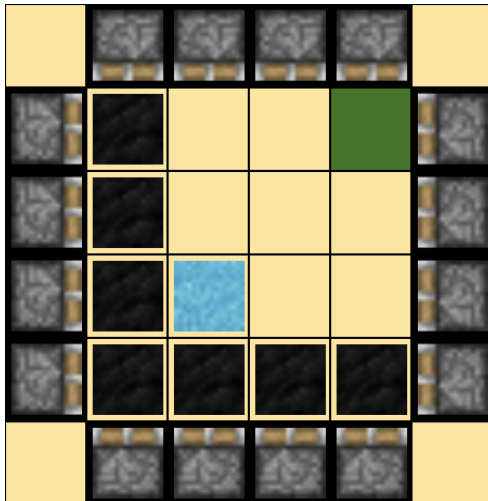
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Objective (MCC 16)



Our strategy should be:

- Not too hard to learn
- Easy to execute
- Quick to execute
- Reliable



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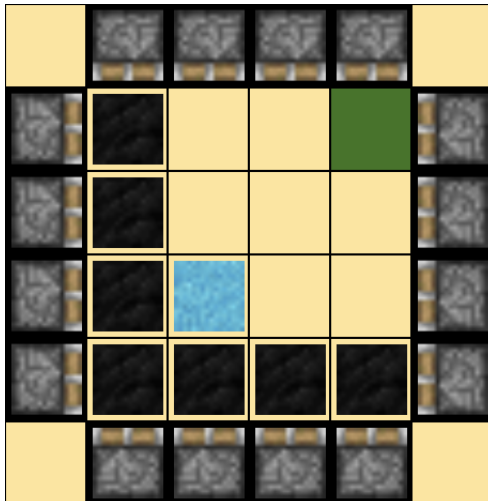
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Objective (MCC 16)



Our strategy should be:

- Not too hard to learn
- Easy to execute
- Quick to execute
- Reliable
- Optimal



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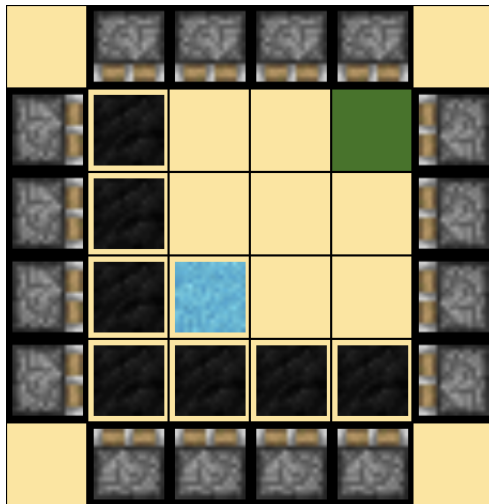
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First, Let's Explore! (MCC 16)



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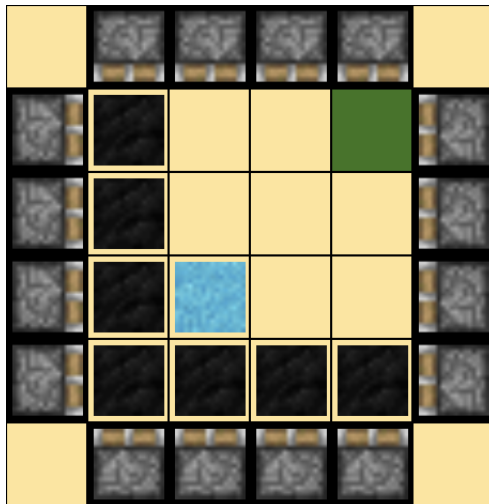
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First, Let's Explore! (MCC 16)



What did we learn?



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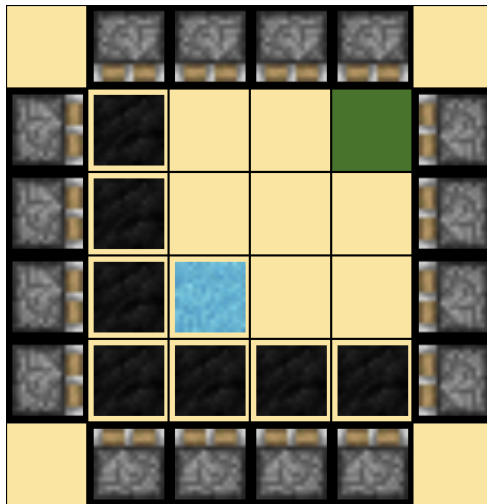
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First, Let's Explore! (MCC 16)



What did we learn?

- To get a block to a corner, we need to use a block in a different corner.



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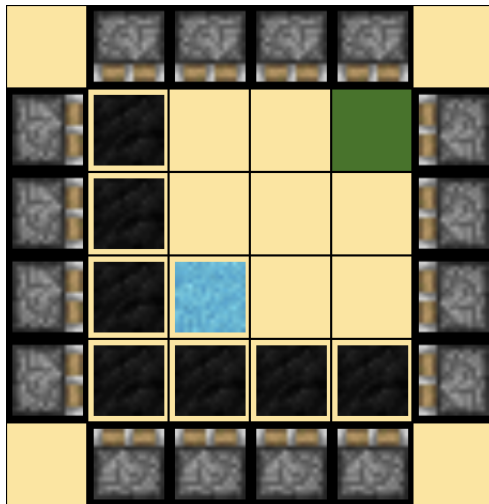
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First, Let's Explore! (MCC 16)



What did we learn?

- To get a block to a corner, we need to use a block in a different corner.
- Therefore, the number of blocks in corners can only decrease!



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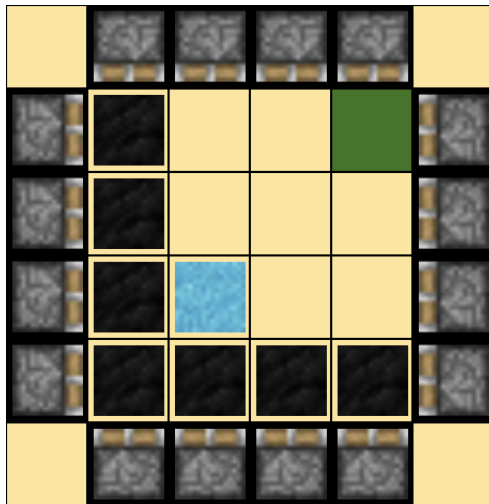


What did we learn?

- To get a block to a corner, we need to use a block in a different corner.
- Therefore, the number of blocks in corners can only decrease!

Definition (Monovariant)

A value that only changes in one direction is called *monovariant*.



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Other Monovariants?



- The # of blocks in the corners of the puzzle **can only decrease**.
- The # of blocks on the (8) edges of the puzzle...
- The # of blocks in the center of the puzzle...

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Other Monovariants?



- The # of blocks in the corners of the puzzle **can only decrease**.
- The # of blocks on the (8) edges of the puzzle...**can increase or decrease**.
- The # of blocks in the center of the puzzle...

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Other Monovariants?



- The # of blocks in the corners of the puzzle **can only decrease**.
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- The # of blocks in the center of the puzzle...**can only increase**.

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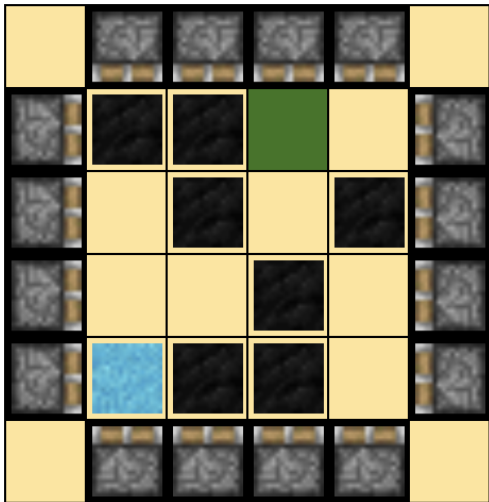
- The # of blocks in the corners of the puzzle **can only decrease**.
- The # of blocks on the (8) edges of the puzzle...**can increase or decrease**.
- The # of blocks in the center of the puzzle...**can only increase**.

Proof (Center Monotonicity).

We'll do it in Minecraft!



A Harder Example (MCC 27)



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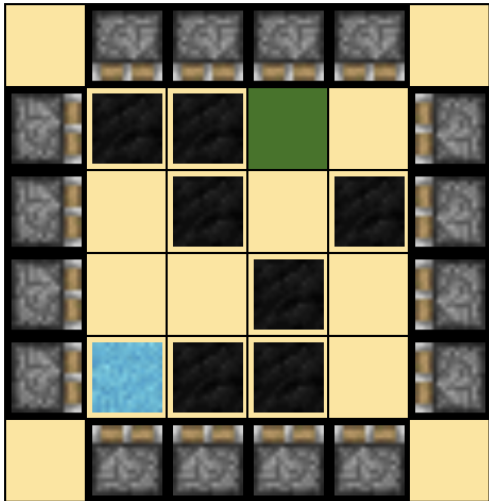
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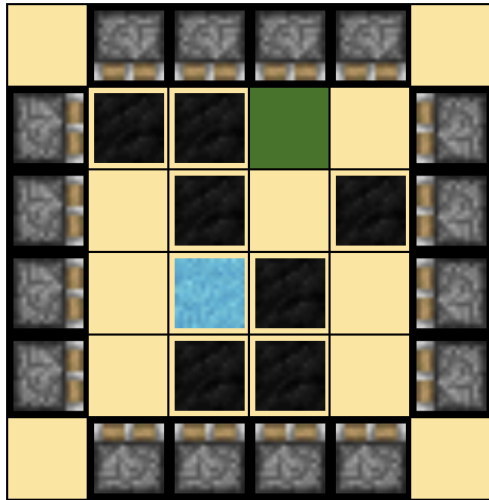
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A Harder Example (MCC 27)



What did we learn?

- If we fill up a row/column in the middle of the board, we'll get stuck!

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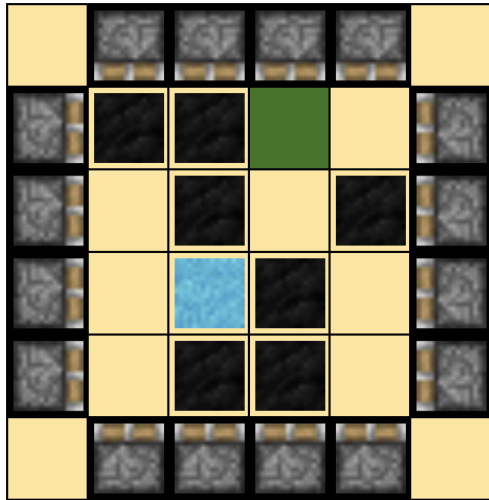
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A Harder Example (MCC 27)



What did we learn?

- If we fill up a row/column in the middle of the board, we'll get stuck!
- If we have 3 blocks in a middle row/column, avoid adding a fourth.

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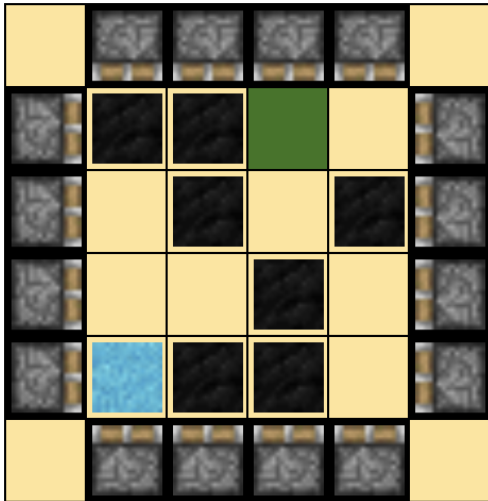
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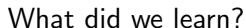
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- If we fill up a row/column in the middle of the board, we'll get stuck!
- If we have 3 blocks in a middle row/column, avoid adding a fourth.
- If we have 3 blocks in all middle rows/columns, we are flexible.

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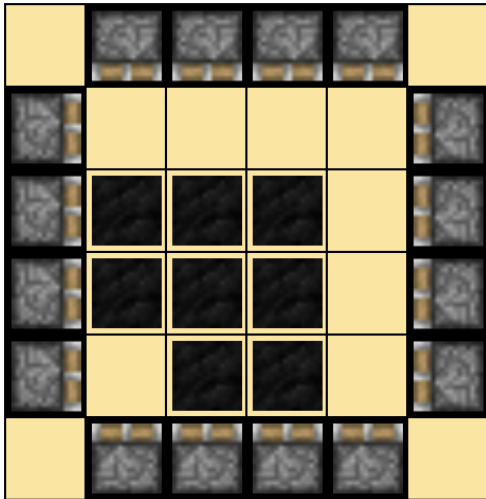
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The 8-Block Square



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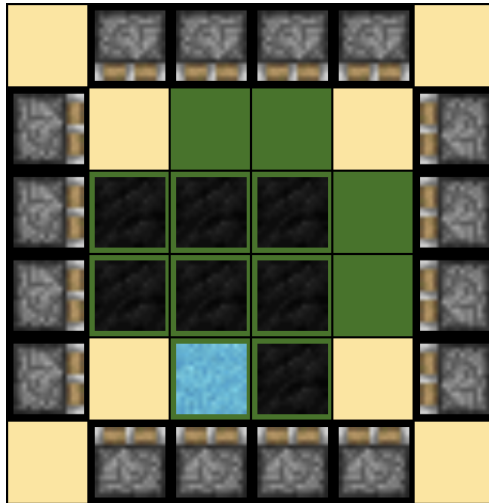
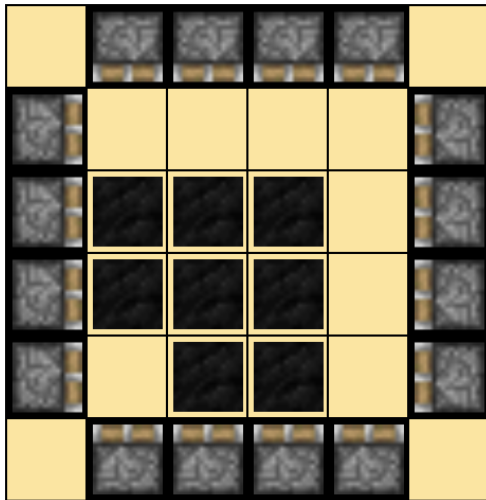
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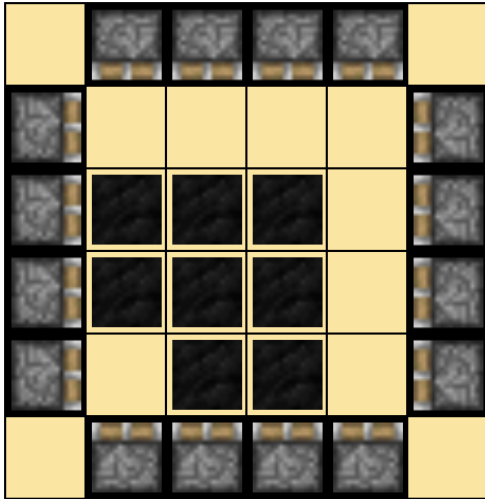
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A Key Property



Definition (Invariant)

We call some property *invariant* when it does not vary.

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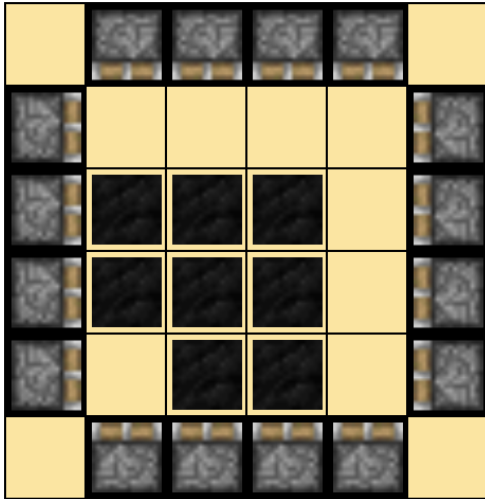
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We call some property *invariant* when it does not vary.

Proof (8-Block Square is Invariant).



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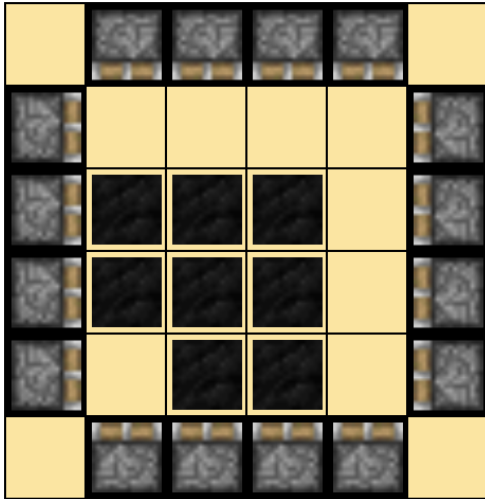
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A Key Property



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We call some property *invariant* when it does not vary.

Proof (8-Block Square is Invariant).

To add a fourth block to any row or column, it has to be added on an edge.



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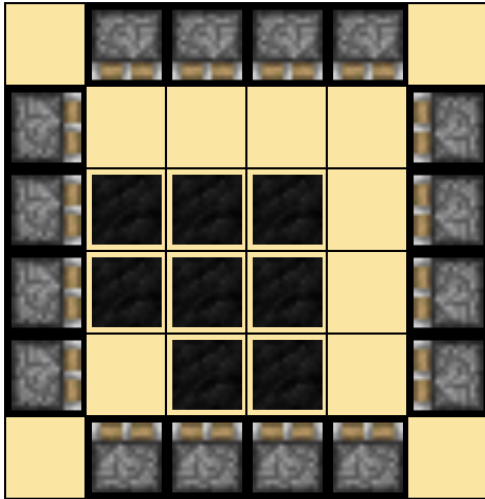
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A Key Property



Definition (Invariant)

We call some property *invariant* when it does not vary.

Proof (8-Block Square is Invariant).

To add a fourth block to any row or column, it has to be added on an edge. We must approach it from one of the sides.



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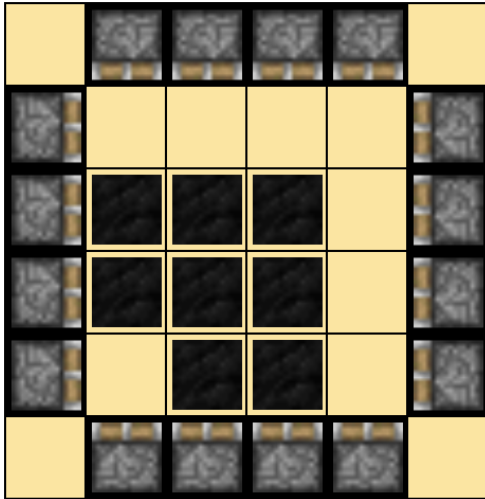
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A Key Property



Definition (Invariant)

We call some property *invariant* when it does not vary.

Proof (8-Block Square is Invariant).

To add a fourth block to any row or column, it has to be added on an edge. We must approach it from one of the sides. But that requires us to push a block from the corner.



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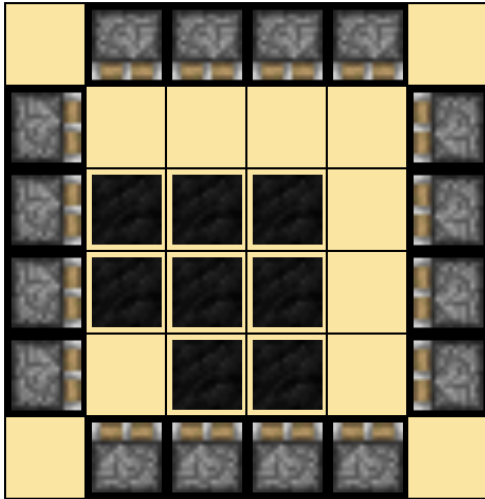
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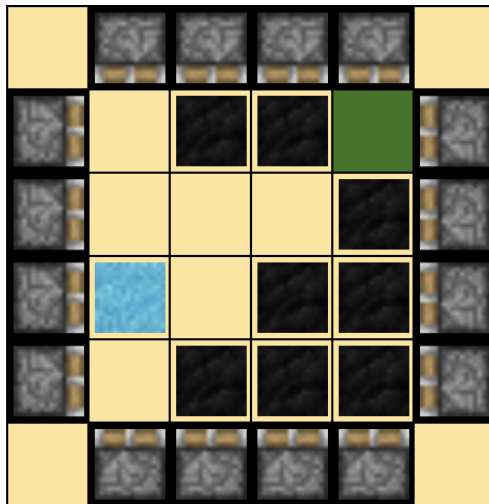
We call some property *invariant* when it does not vary.

Proof (8-Block Square is Invariant).

To add a fourth block to any row or column, it has to be added on an edge. We must approach it from one of the sides. But that requires us to push a block from the corner. And if there's a block in the corner, then we never have to move it!



Challenge Puzzle (MCC 22)



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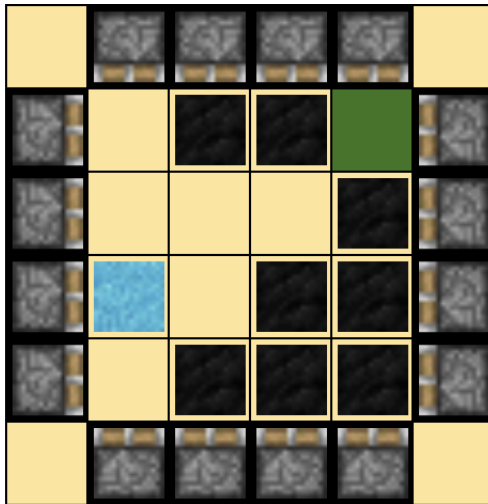
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Challenge Puzzle (MCC 22)



What did we learn?



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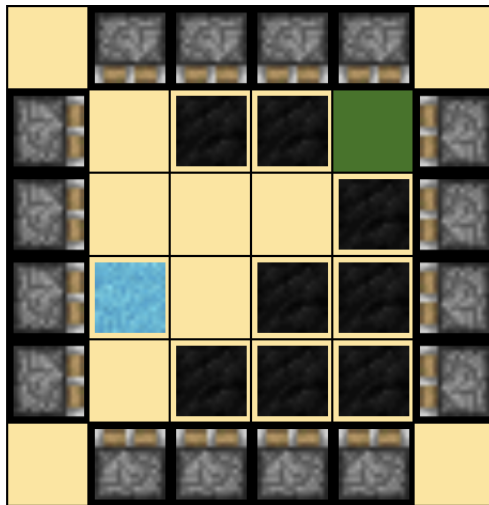
Conclusion

Challenge Puzzle (MCC 22)



What did we learn?

- Combined with one block in a corner, the 8-Block Square can get us anywhere!



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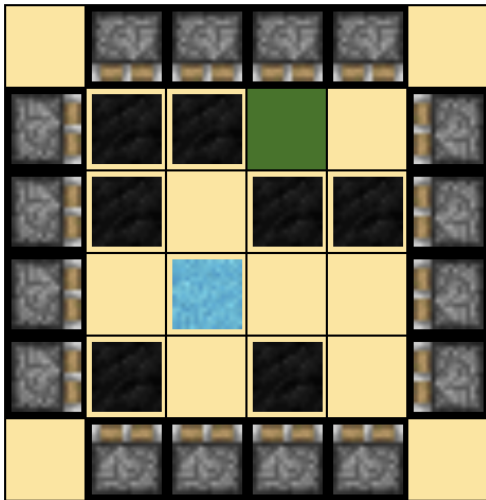
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Some More Challenge Puzzles (Bonus 1/Bonus 2)



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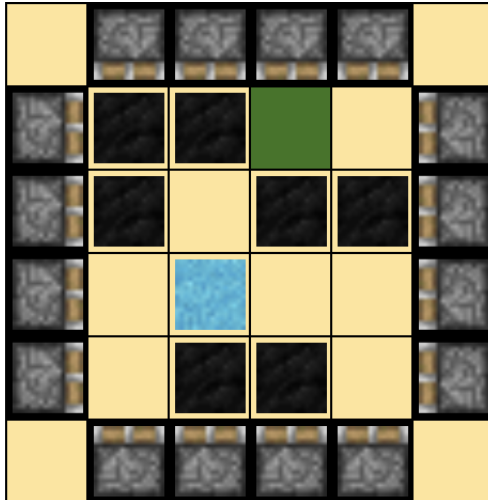
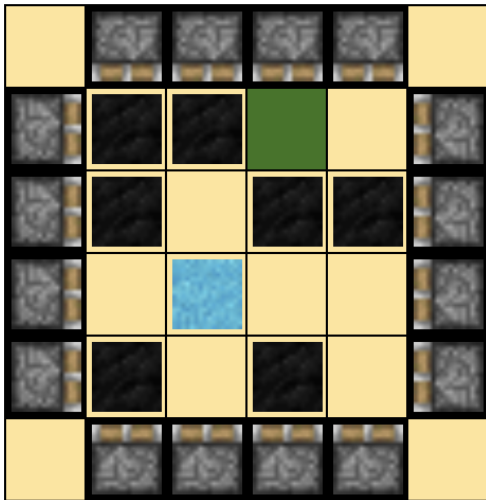
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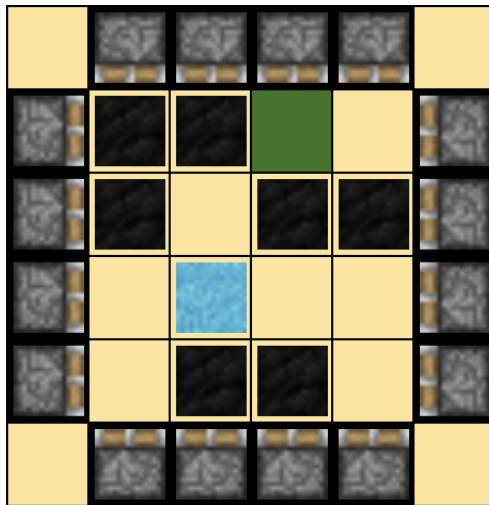
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We've got an issue here! (Bonus 2)



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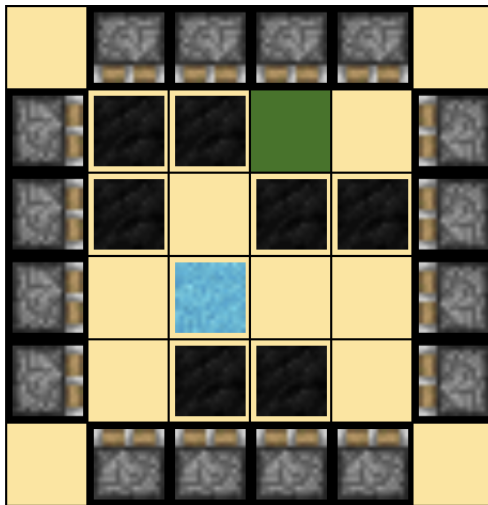
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We've got an issue here! (Bonus 2)



- If we try to collapse the second row, we fill up the second column.



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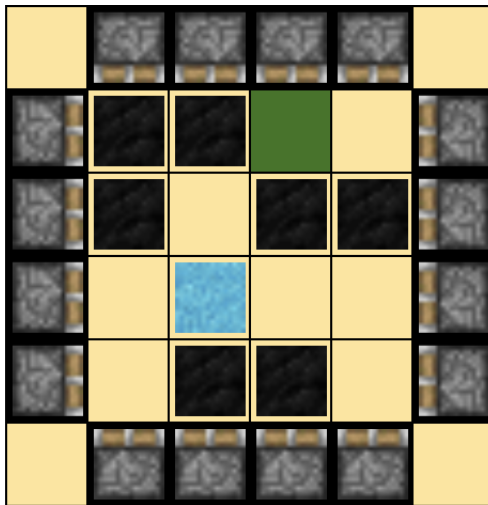
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We've got an issue here! (Bonus 2)



- If we try to collapse the second row, we fill up the second column.
- If we try to collapse the second column, we fill up the second row.



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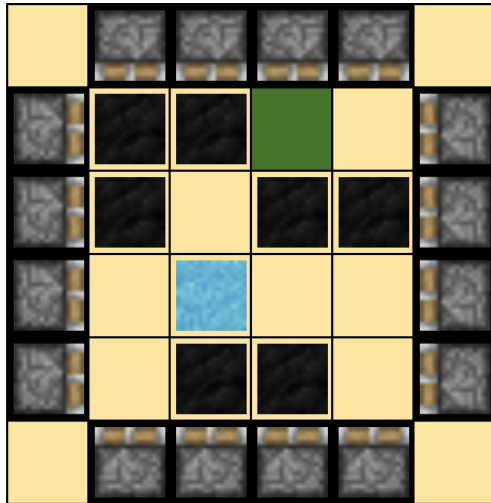
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We've got an issue here! (Bonus 2)

- If we try to collapse the second row, we fill up the second column.
- If we try to collapse the second column, we fill up the second row.
- We can prove that there is no way to get the 8-Block square!



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Let's Recap!



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Let's Recap!

- Corner blocks mostly matter if the goal is in the corner.



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Let's Recap!



- Corner blocks mostly matter if the goal is in the corner.
- If there are several blocks, we can usually make an 8-block square.

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Let's Recap!



- Corner blocks mostly matter if the goal is in the corner.
- If there are several blocks, we can usually make an 8-block square.
- Sometimes, we get stuck!

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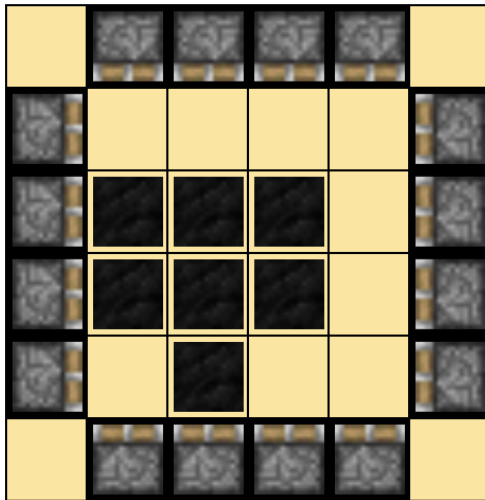
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Let's Recap! (7 Blocks)



- Corner blocks mostly matter if the goal is in the corner.
- If there are several blocks, we can usually make an 8-block square.
 - **What if we have fewer blocks?**
- Sometimes, we get stuck!



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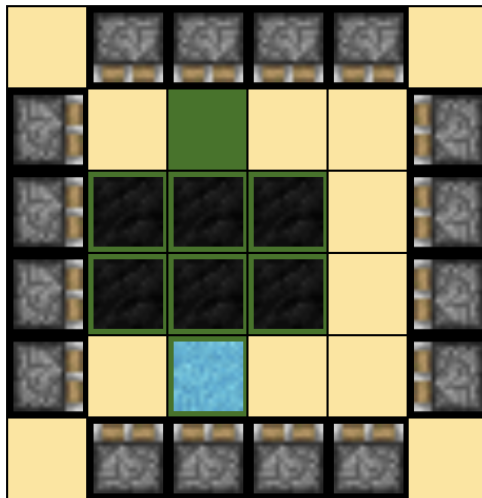
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Let's Recap! (7 Blocks)

- Corner blocks mostly matter if the goal is in the corner.
- If there are several blocks, we can usually make an 8-block square.
 - **What if we have fewer blocks?**
- Sometimes, we get stuck!



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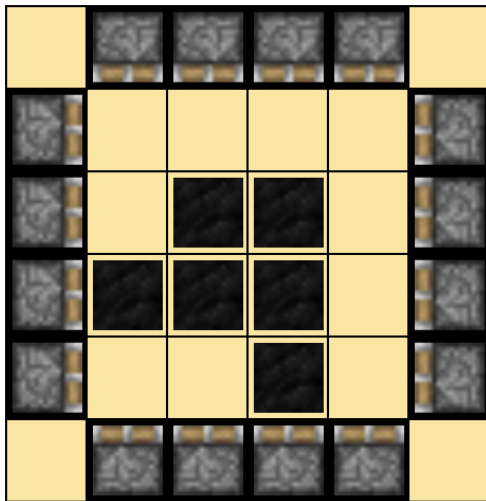
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Let's Recap! (6 Blocks)



- Corner blocks mostly matter if the goal is in the corner.
- If there are several blocks, we can usually make an 8-block square.
 - **What if we have fewer blocks?**
- Sometimes, we get stuck!



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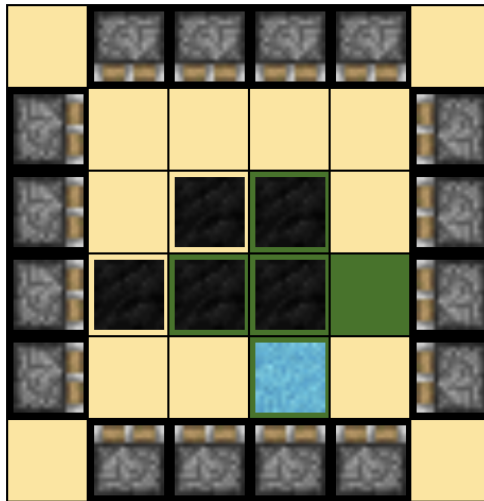
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Let's Recap! (6 Blocks)

- Corner blocks mostly matter if the goal is in the corner.
- If there are several blocks, we can usually make an 8-block square.
 - **What if we have fewer blocks?**
- Sometimes, we get stuck!



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Let's Recap! (Bad Pattern)



- Corner blocks mostly matter if the goal is in the corner.
- If there are several blocks, we can usually make an 8-block square.
 - What if we have fewer blocks?
- Sometimes, we get stuck!
 - **When can we avoid this?**

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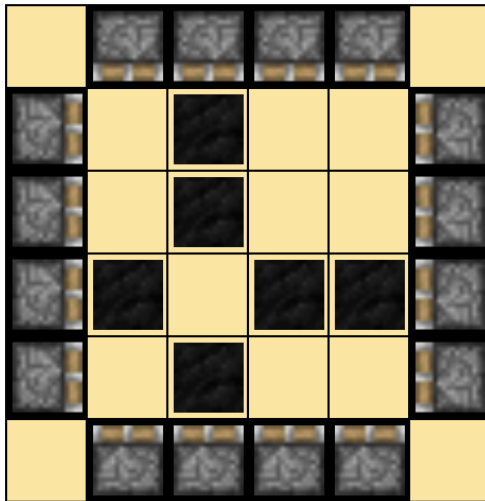
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Let's Recap! (Bad Pattern)



- Corner blocks mostly matter if the goal is in the corner.
- If there are several blocks, we can usually make an 8-block square.
 - What if we have fewer blocks?
- Sometimes, we get stuck!
 - **When can we avoid this?**



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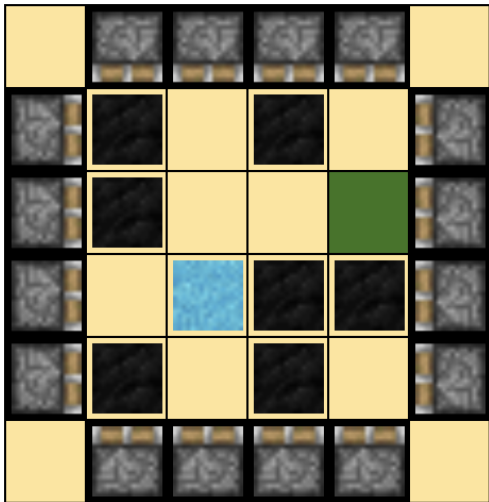
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The 'Best' Strategy?

What Makes a Good Strategy? (MCC 21)



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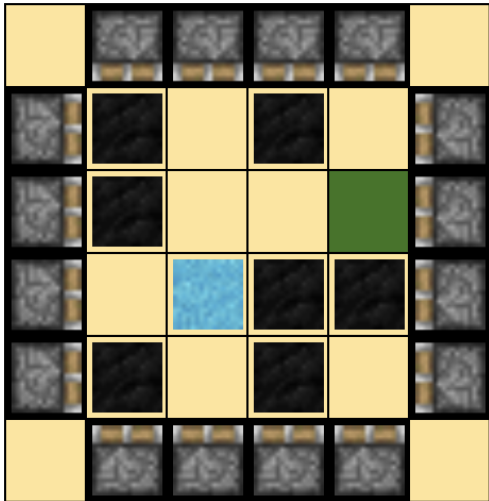
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What Makes a Good Strategy? (MCC 21)



- Not too hard to learn?

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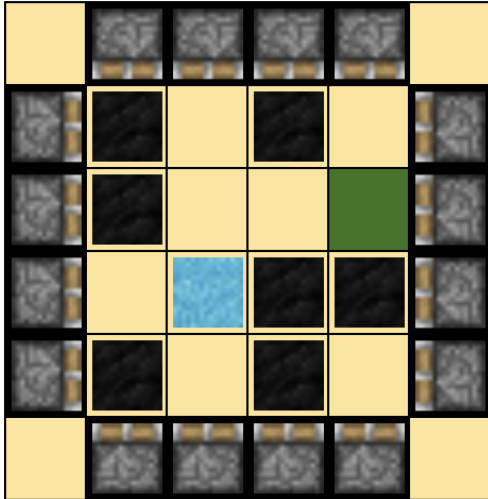
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What Makes a Good Strategy? (MCC 21)



- Not too hard to learn
- Easy to execute?

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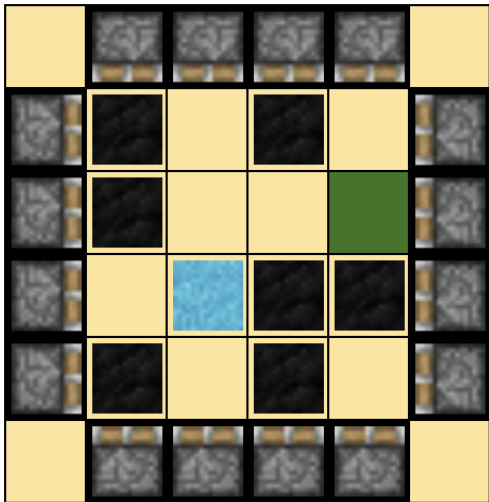
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What Makes a Good Strategy? (MCC 21)



- Not too hard to learn
- Easy to execute
- Quick to execute?

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Procedure:

- Do we need a corner block?



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Procedure:

- Do we need a corner block?
- If we can, form the 8-Block Square.



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Procedure:



- Do we need a corner block?
- If we can, form the 8-Block Square.
 - Collapse dangerous rows first!

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Procedure:



- Do we need a corner block?
- If we can, form the 8-Block Square.
 - Collapse dangerous rows first!
- We rely on the same kind of four-move sequence.

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Procedure:



- Do we need a corner block?
- If we can, form the 8-Block Square.
 - Collapse dangerous rows first!
- We rely on the same kind of four-move sequence.
- Shift into the corner, if needed.

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Procedure:



- Do we need a corner block?
- If we can, form the 8-Block Square.
 - Collapse dangerous rows first!
- We rely on the same kind of four-move sequence.
- Shift into the corner, if needed.
- **Not much forethought needed!**

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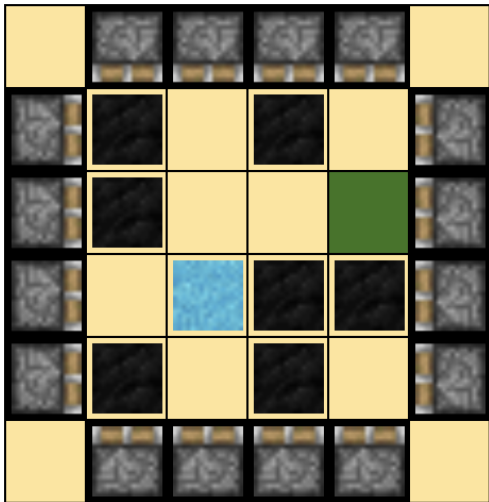
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What Makes a Good Strategy? (MCC 21)



- Not too hard to learn
- Easy to execute
- Quick to execute
- Reliable?

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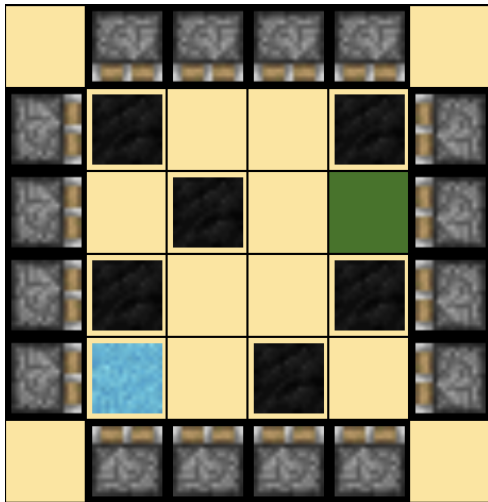
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Can we solve most puzzles this way? (MCC 15/MCC 18)



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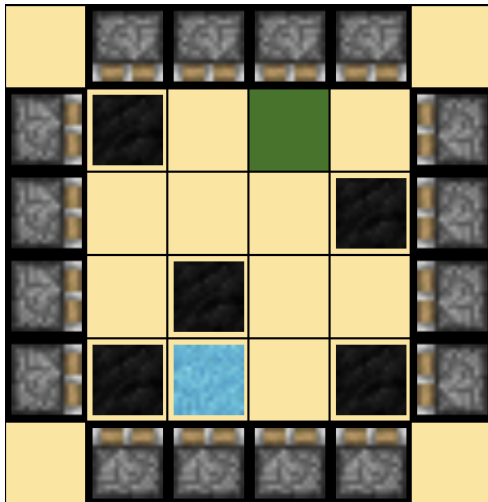
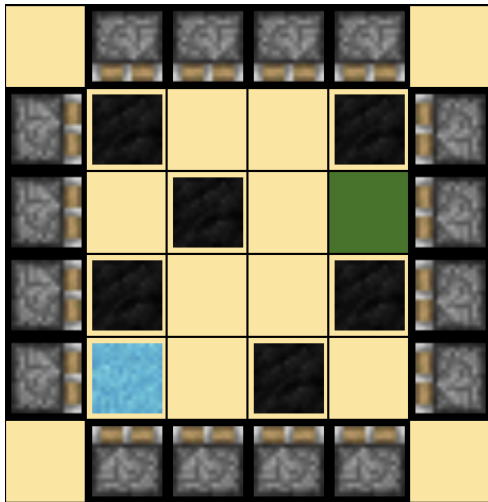
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Can we solve most puzzles this way? (MCC 15/MCC 18)



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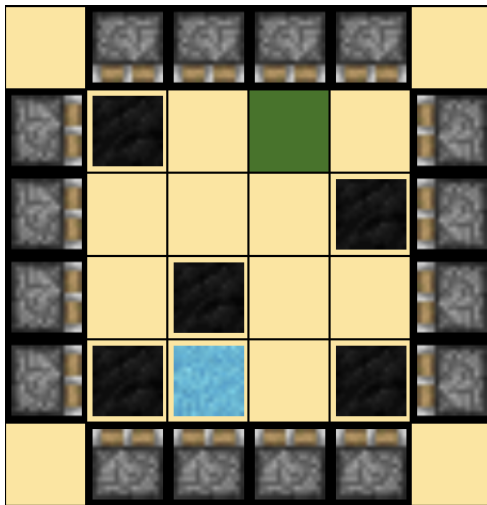
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Can we solve most puzzles this way? (MCC 18)



How many puzzles use the 8-block square?



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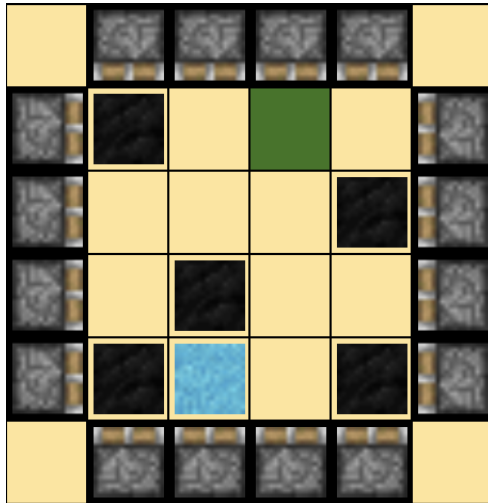
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Can we solve most puzzles this way? (MCC 18)

How many puzzles use the 8-block square?

- At least 8 blocks



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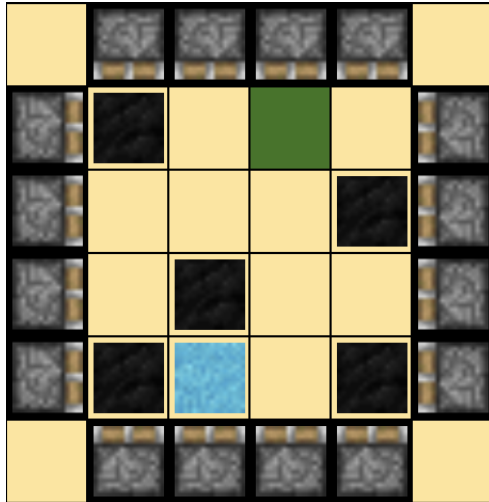
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Can we solve most puzzles this way? (MCC 18)

How many puzzles use the 8-block square?

- At least 8 blocks
- No middle wall/bad pattern!



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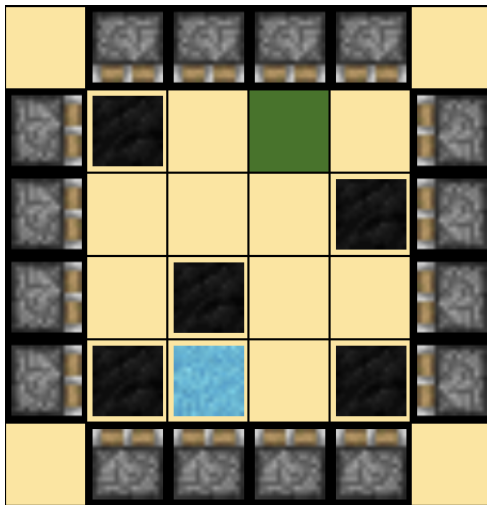
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Can we solve most puzzles this way? (MCC 18)



How many puzzles use the 8-block square?

- At least 8 blocks
- No middle wall/bad pattern!
- At most 8 blocks not in corners



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- At least 8 blocks
- No middle wall/bad pattern!
- At most 8 blocks not in corners

Conclusion



Can we solve most puzzles this way? (MCC 18)

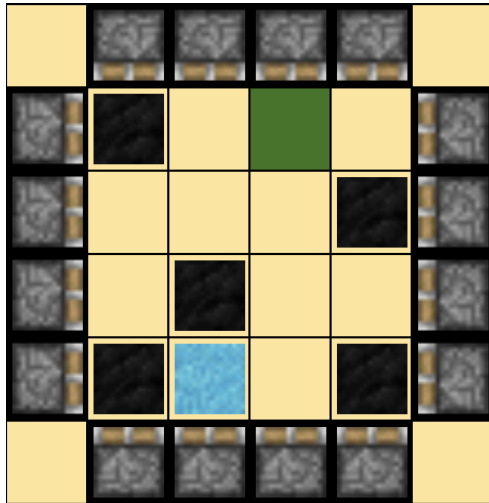
How many puzzles use the 8-block square?

- At least 8 blocks
- No middle wall/bad pattern!
- At most 8 blocks not in corners

Pretty tricky to count...

Do we care about symmetry?

We can't be deceived!



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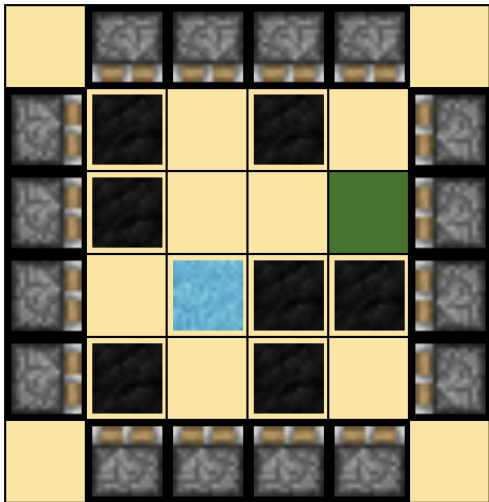
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What Makes a Good Strategy? (MCC 21)



- Not too hard to learn
- Easy to execute
- Quick to execute
- Reliable (in some ways)
- Optimal?

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Why Can't a Computer Just Solve This?



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Why Can't a Computer Just Solve This?



- To brute force this, we'd have to consider all possible moves from any of $\approx 4 \cdot 15 \cdot 2^{14}$ starting positions.

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Why Can't a Computer Just Solve This?



- To brute force this, we'd have to consider all possible moves from any of $\approx 4 \cdot 15 \cdot 2^{14}$ starting positions.
- We can make some smart decisions to narrow down our search:

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Why Can't a Computer Just Solve This?

- To brute force this, we'd have to consider all possible moves from any of $\approx 4 \cdot 15 \cdot 2^{14}$ starting positions.
- We can make some smart decisions to narrow down our search:
 - Greedy algorithm?

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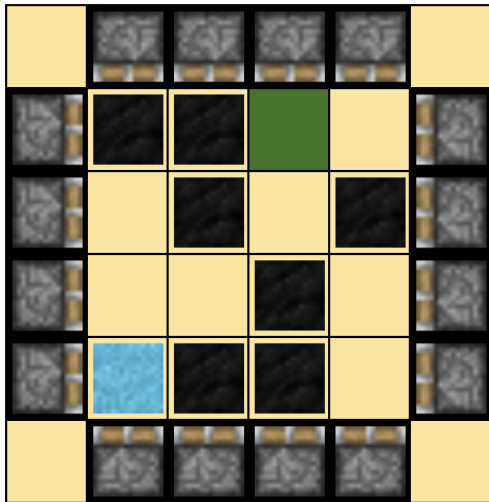
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Beware the Greedy Algorithm! (MCC 27)



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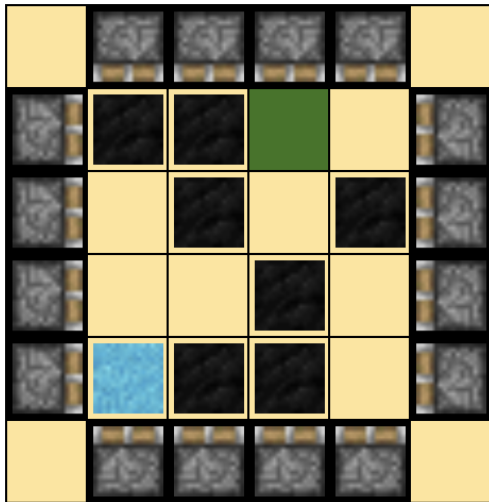
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Beware the Greedy Algorithm! (MCC 27)



Definition (Greedy Algorithm)

An algorithm that tries to make a “locally optimal” move at each step.

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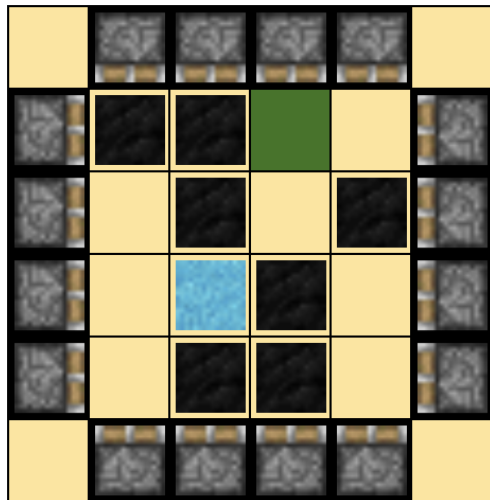
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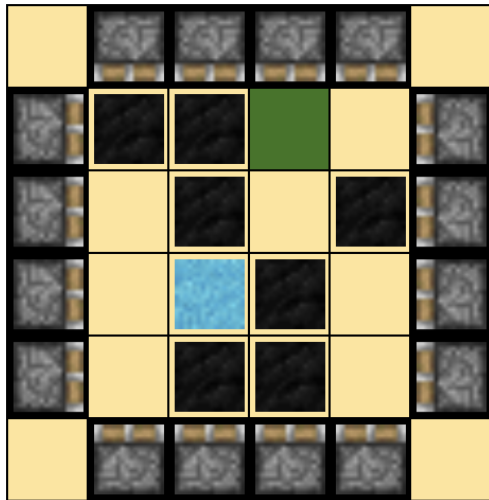
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- Reddit user [/u/ctladvance](#) created a solver which uses this type of strategy to search for a solution in the fewest number of moves.

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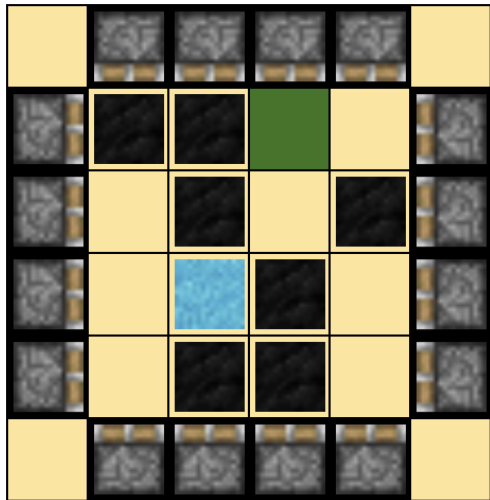
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- Reddit user [/u/ctladvance](#) created a solver which uses this type of strategy to search for a solution in the fewest number of moves.
- Since the puzzle is small, this search was usually fast!

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Why Can't a Computer Just Solve This?



- To brute force this, we'd have to consider all possible moves from any of $\approx 4 \cdot 15 \cdot 2^{14}$ starting positions.
- We can make some smart decisions to narrow down our search:
 - Greedy algorithm?
 - Tablebase

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Tablebase



Definition (Tablebase)

A *tablebase* is a database with many possible positions. Position A is linked to Position B if a legal move can take you from A to B.

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- Example: Chess (up to 8 pieces)



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- Example: Chess (up to 8 pieces)
- White to move and win



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Definition (Tablebase)

A *tablebase* is a database with many possible positions. Position A is linked to Position B if a legal move can take you from A to B.

- Example: Chess (up to 8 pieces)
- White to move and win
- White has a move such that all possible moves for black are into losing positions.



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Definition (Tablebase)

A *tablebase* is a database with many possible positions. Position A is linked to Position B if a legal move can take you from A to B.

- Example: Chess (up to 8 pieces)
- White to move and win
- White has a move such that all possible moves for black are into losing positions. **Ra6!**



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Tablebase



Definition (Tablebase)

A *tablebase* is a database with many possible positions. Position A is linked to Position B if a legal move can take you from A to B.

- New positions can be assessed by tracing back from all known positions, so we can do something similar with our puzzles!



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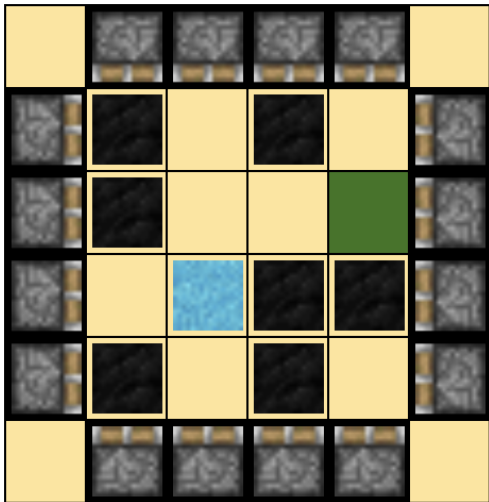
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What Makes a Good Strategy? (MCC 21)



- **Not too hard to learn**
- **Easy to execute**
- **Quick to execute**
- **Reliable** (in some ways)
- **Not Optimal** (but that's okay!)

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Further Questions



- What if we have a middle wall/bad pattern? (Bonus 2/Bonus 3/Bonus 4)
- What if we have too few blocks for the 8-block square to be useful? (MCC 18)
- How many puzzles are solved using the 8-block square strategy?
- How many total puzzles are there (up to symmetry)?
- What if we looked at a bigger version of this puzzle?

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Thank You!



- Noxcrew (the creators of Sands of Time)
- All the Minecraft Championship players who've tried this puzzle
- Reddit user /u/ctladvance (who wrote some programs to solve this puzzle)
- The Summer of Math Exposition organizers (without whom this project wouldn't exist)
- Dr. Heather Muchowski, Westminster College

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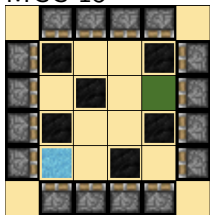
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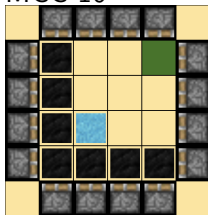
Lots of Puzzles!



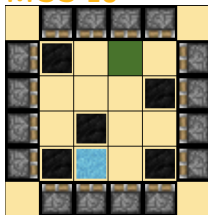
MCC 15



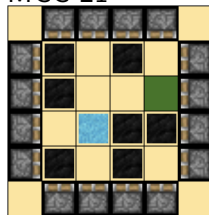
MCC 16



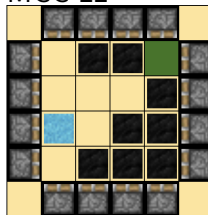
MCC 18



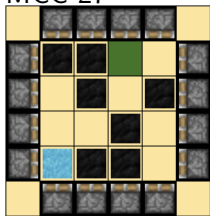
MCC 21



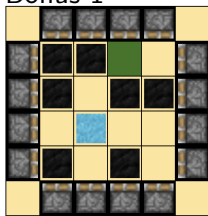
MCC 22



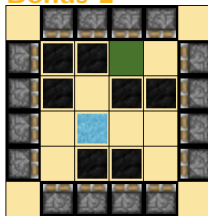
MCC 27



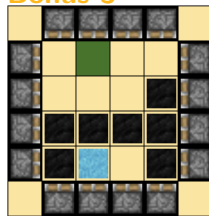
Bonus 1



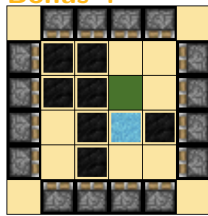
Bonus 2



Bonus 3



Bonus 4



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